Exact gauge fields from anti-de Sitter space (2301.03606)

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- For us the strategy would be: $\mathcal{I} \times \mathrm{Ad}S_3 \xleftarrow{\text{conformal}} \mathrm{Ad}S_4 \xrightarrow{\text{conformal}} \mathcal{I} \times S^3_+$ followed by the previous $\mathcal{I} \times S^3 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$ and noting that $\mathrm{Ad}S_3$ is the group manifold of SU(1,1).

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- In both cases, gluing of two copies of $\mathrm{d}S_4/\mathrm{Ad}S_4$ is used to cover the full Minkowski space.

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Recap: $\mathrm{dS}_4 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$

It is well known that dS_4, viewed through embedding in $\mathbb{R}^{1,4}$ via

$$-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = R^2 \;, \tag{1}$$

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$$\begin{aligned}\mathrm{d}s^2 &= \frac{R^2}{\sin^2\tau}\left(-\mathrm{d}\tau^2+\mathrm{d}\chi^2+\sin^2\chi\mathrm{d}\Omega_2^2\right) = \frac{R^2}{\sin^2\tau}\left(\mathrm{d}\tau^2+\mathrm{d}\Omega_3^2\right)\\ &= \frac{R^2}{t^2}\left(-\mathrm{d}t^2+\mathrm{d}r^2+r^2\mathrm{d}\Omega_2^2\right) = \frac{R^2}{t^2}\left(-\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right)\end{aligned} \tag{2}$$

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where $\tau\in\mathcal{I}:=(\pi/2,\pi/2)$, $\chi\in[0,\pi]$, $t\in\mathbb{R}_+$ & $r\in\mathbb{R}$. Notice that this only covers future half of the Minkowski space! This is due to the following constraint:

$$\cos\tau>\cos\chi \quad\Longleftrightarrow\quad \chi>|\tau|\ . \tag{3}$$

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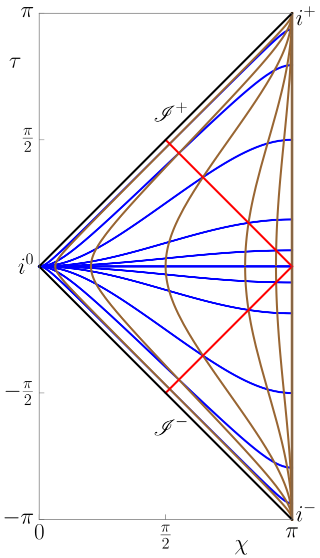
- This is clearly deomnstrated with the (τ,χ) Penrose diagram on the right with t- and r-slices.



$\mathrm{AdS}_4 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$

We can isometrically embed AdS_4 inside $\mathbb{R}^{2,3}$ as

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Here also this takes two useful conformal avatars, easily seen from its flat-metric:

$$\begin{aligned} \mathrm{d}s^2 \;&=\; \frac{R^2}{\cos^2\psi}\big(\mathrm{d}\psi^2 - \cosh^2\!\rho\,\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\!\rho\,\mathrm{d}\phi^2\big) \;=\; \frac{R^2}{\cos^2\psi}\big(\mathrm{d}\psi^2 + \mathrm{d}\Omega^2_{1,2}\big) \\ &=\; \frac{R^2}{\cos^2\chi}\big(-\mathrm{d}\tau^2 + \mathrm{d}\chi^2 + \sin^2\!\chi\,\mathrm{d}\Omega^2_2\big) \;=\; \frac{R^2}{\cos^2\chi}\big(-\mathrm{d}\tau^2 + \mathrm{d}\Omega^2_{3+}\big) \end{aligned} \tag{6}$$

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where $\psi\in\mathcal{I}\equiv(-\pi/2,\pi/2)$, $\rho\in\mathbb{R}_+$, $\phi,\tau\in S^1$, but $\chi\in[0,\pi/2]$. Thus, we get cylinders

(a) over (half of) the 3-sphere S^3, and

(b) over $\underline{\mathrm{AdS}_3\cong\mathrm{SU}(1,1)/\{\pm\mathrm{id}\}}$ with metric $\mathrm{d}\Omega_{1,2}$.

Here the S^3_+-structure is, in fact, nothing but conformal compactification of H^3. To see this, let us change coordinates $(\rho,\psi) \to (\lambda,\theta) \in \mathbb{R}_+ \times [0,\pi]$ in (6) via

$$\begin{aligned} \tanh\rho &= \sin\theta\,\tanh\lambda \qquad \text{and} \qquad \tan\psi = -\cos\theta\,\sinh\lambda \\ \implies \quad \mathrm{d}s^2 &= R^2\big(-\cosh^2\lambda\,\mathrm{d}\tau^2 + \mathrm{d}\lambda^2 + \sinh^2\lambda\,\mathrm{d}\Omega_2^2\big)\ . \end{aligned} \tag{7}$$

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$$\begin{aligned} &\tanh\rho = \sin\theta \, \tanh\lambda = \varepsilon \, \sin\theta \, \sin\chi \quad \& \quad \tan\psi = -\varepsilon \, \cos\theta \, \sinh\lambda = -\varepsilon \, \cos\theta \, \tan\chi \\ &\text{where} \begin{cases} \varepsilon = +1 \; : \quad \rho, \lambda \in \mathbb{R}_+ \, , \;\; \chi \in [0, \tfrac{\pi}{2}) \quad \Leftrightarrow \quad \text{northern hemisphere } S^3_+ \\ \varepsilon = -1 \; : \quad \rho, \lambda \in \mathbb{R}_- \, , \;\; \chi \in (\tfrac{\pi}{2}, \pi] \quad \Leftrightarrow \quad \text{southern hemisphere } S^3_- \end{cases} \end{aligned} \tag{8}$$

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- Use the previous $(\tau,\chi)\to(t,r)$ map to get to the Minkowski space.

This gluing, unlike in the de Sitter case, is not smooth and has singularity at the boundary:

$$\begin{aligned} \{|\psi|=\tfrac{\pi}{2}\} \;&=\; \{\lambda=\pm\infty\} \;=\; \{\chi=\tfrac{\pi}{2}\} \\ \Longleftrightarrow \qquad \{r^2-t^2=R^2\} \;&=:\; H_R^{1,2} \;\cong\; \mathrm{dS}_3 \;\cong\; \mathcal{I}_\tau \times S^2\big|_{\mathrm{bdy}} \;. \end{aligned} \tag{9}$$

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One needs to be careful of the orientation of two two copies at the boundary as shown below

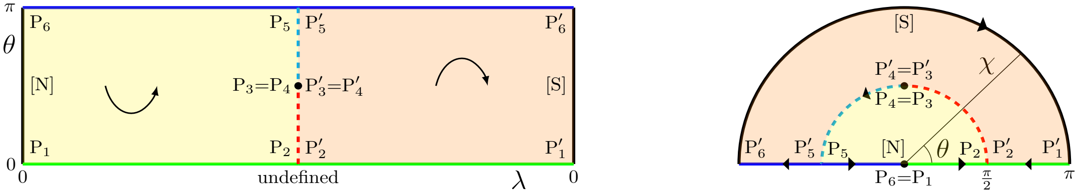
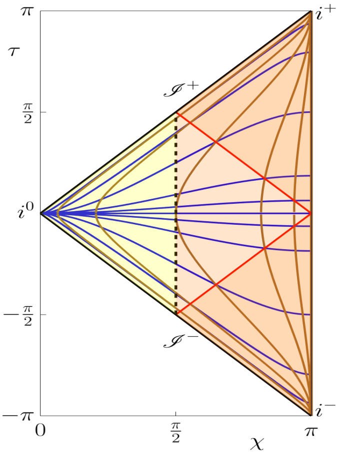


Figure 1: Gluing S_3^+ (yellow shaded region) to S_3^- (orange shaded region) along the (dashed) boundary.



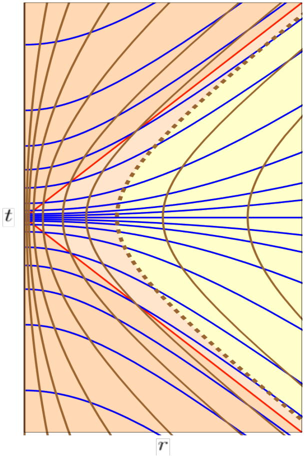


Figure 2: Gluing of two AdS_4 to reveal the full Minkowski space with the lightcone (red). Left: (τ, χ) AdS_4 space (two copies) yielding the Penrose diagram with constant t- (blue) and r-slices (brown). Right: (t, r) Minkowski space with boundary hyperbola $H_R^{1,2}$ (dashed) and constant τ- (blue) and χ-slices (brown).

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SU(1, 1) Lie algebra and one-forms

We start by noticing that AdS_3 is the group manifold of SU(1, 1):

$$g: \ \mathrm{AdS}_3 \to \mathrm{SU}(1,1) \qquad \text{via} \qquad (y^1, y^2, y^3, y^4) \mapsto \begin{pmatrix} y^1 - \mathrm{i}y^2 & y^3 - \mathrm{i}y^4 \\ y^3 + \mathrm{i}y^4 & y^1 + \mathrm{i}y^2 \end{pmatrix} . \tag{10}$$

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This map also yields the left-invariant one-forms $e^\alpha,\ \alpha=0,1,2$ via Maurer–Cartan method:

$$\Omega_L(g) = g^{-1}\mathrm{d}g = e^\alpha I_\alpha\ ; \quad I_0 = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix},\ I_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ I_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \tag{11}$$

where the $\mathfrak{sl}(2,\mathbb{R})$ generators I_α of $\mathrm{PSL}(2,\mathbb{R}) \cong \mathrm{SU}(1,1)/\{\pm\mathrm{id}\}$ are subject to

$$[I_\alpha, I_\beta] = 2\,f^{\gamma}_{\ \alpha\beta}\, I_\gamma \qquad \text{and} \qquad \mathrm{tr}(I_\alpha\, I_\beta) = 2\,\eta_{\alpha\beta} \qquad \text{with} \quad (\eta_{\alpha\beta}) = \mathrm{diag}(-1,1,1)\ . \tag{12}$$

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Explicitly, we get the following one-forms on AdS_3

$$\begin{aligned} e^0 &= \cosh^2\rho\, \mathrm{d}\tau + \sinh^2\rho\, \mathrm{d}\phi , \\ e^1 &= \cos(\tau-\phi)\, \mathrm{d}\rho + \sinh\rho\ \cosh\rho\ \sin(\tau-\phi)\, \mathrm{d}(\tau+\phi) , \\ e^2 &= -\sin(\tau-\phi)\, \mathrm{d}\rho + \sinh\rho\ \cosh\rho\ \cos(\tau-\phi)\, \mathrm{d}(\tau+\phi) . \end{aligned} \tag{13}$$

The (non-)equivariant ansatz

These one-forms e^α satisfy following Cartan structure equation

$$\mathrm{d}e^\alpha + f^\alpha_{\ \beta\gamma}\, e^\beta \wedge e^\gamma = 0 \tag{14}$$

and provide a local orthonormal frame on the cylinder $\mathcal{I} \times \mathrm{AdS}_3$:

$$\mathrm{d}s^2_{\mathrm{cyl}} \;=\; \mathrm{d}\psi^2 + \eta_{\alpha\beta}\, e^\alpha e^\beta \;=:\; (e^\psi)^2 - (e^0)^2 + (e^1)^2 + (e^2)^2 \;. \tag{15}$$

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To make gauge field $\mathcal{A} = \mathcal{A}_\psi\, e^\psi + \mathcal{A}_\alpha\, e^\alpha$ here SU(1,1)-symmetric we set $\mathcal{A}_\psi = 0$, such that

$$\mathcal{A} \;=\; X_\alpha(\psi)\, e^\alpha \qquad \Longrightarrow \qquad \mathcal{F} \;=\; X_\alpha'\, e^\psi \wedge e^\alpha + \tfrac{1}{2}\big(-2f^\alpha_{\beta\gamma} X_\alpha + [X_\beta, X_\gamma]\big)\, e^\beta \wedge e^\gamma\ . \tag{16}$$

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Moreover, to satisfy the Gauss-law constraint $[X_\alpha, X^{'}_\alpha] = 0$ we chose

$$X_0 = \Theta_0(\psi)\, I_0 \ , \quad X_1 = \Theta_1(\psi)\, I_1 \ , \quad X_2 = \Theta_2(\psi)\, I_2 \ . \tag{17}$$

This gives us the following Yang–Mills Langrangian

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{4}\mathrm{tr}\mathcal{F}_{\psi\alpha}\mathcal{F}^{\psi\alpha} + \tfrac{1}{8}\mathrm{tr}\mathcal{F}_{\beta\gamma}\mathcal{F}^{\beta\gamma} \\ &= \tfrac{1}{2}\{(\Theta_1')^2 + (\Theta_2')^2 + (\Theta_0')^2\} \; - \; 2\{(\Theta_1 - \Theta_2\Theta_0)^2 + (\Theta_2 - \Theta_0\Theta_1)^2 + (\Theta_0 - \Theta_1\Theta_2)^2\} \; ,\end{aligned} \tag{18}$$

which enjoys a discrete symmetry of the permutation group S_4.

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$$
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\end{aligned}
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which enjoys a discrete symmetry of the permutation group S_4. Two subgroups of S_4, namely S_3 (non-Abelian) and D_8 (Abelian), yields interesting solutions as follows.

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- **Non-equivariant Abelian ansatz:**

$$\Theta_0 = \mathrm{h}(\psi) \qquad \text{while} \qquad \Theta_1 = \Theta_2 = 0 \,,$$
$$\Longrightarrow \mathrm{h}'' = -4\,\mathrm{h}$$

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- **Equivariant non-Ablian ansatz:**

$$\Theta_0 = \Theta_1 = \Theta_2 \ =: \ \tfrac{1}{2}\big(1+\Phi(\psi)\big)$$
$$\implies \Phi'' \ = \ 2\,\Phi\,(1-\Phi^2) \ = \ -\frac{\partial V}{\partial \Phi}$$

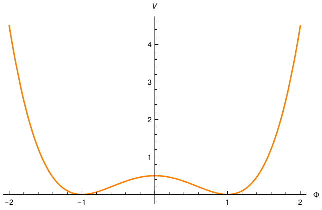

Figure 3: Potential $V(\Phi) = \frac{1}{2}(\Phi^2-1)^2$.

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Non-Abelian solution

Using $(\tau,\rho,\psi) \to (\tau,\chi,\theta) \to (t,r,\theta)$ for $R{=}1$ and abbreviations $x \cdot \mathrm{d}x := x_\mu \mathrm{d}x^\mu$ and $\varepsilon^0 := 1$, $\varepsilon^1 = \varepsilon^2 := \varepsilon$, we can write the Minkowski one-forms in a compact form as,

$$\begin{aligned} e^\alpha &= \tfrac{\varepsilon^\alpha}{\lambda^2+4z^2} \big(2(\lambda{+}2)\,\mathrm{d}x^\alpha - 4x^\alpha\, x\cdot\mathrm{d}x - 4\, f^{\alpha}_{\ \beta\gamma}\, x^\beta \mathrm{d}x^\gamma\big)\ , \\ e^\psi &= \tfrac{\varepsilon}{\lambda^2+4z^2} \big(-2\lambda\,\mathrm{d}z + 4z\, x\cdot\mathrm{d}x\big)\ , \qquad \text{where} \quad \lambda := r^2 - t^2 - 1\ . \end{aligned} \tag{19}$$

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e^\psi &= \tfrac{\varepsilon}{\lambda^2+4z^2}\left(-2\lambda\,\mathrm{d}z + 4z\, x\cdot\mathrm{d}x\right)\ , \qquad \text{where}\quad \lambda\ :=\ r^2-t^2-1\ .
\end{aligned} \tag{19}
$$

Fields are obtained from the field strength of $\mathcal{A}\equiv A=\frac{1}{2}\Big(1+\Phi\big(\psi(x)\big)\Big)\, I_\alpha\; e^\alpha_{\ \mu}\;\mathrm{d}x^\mu$,

$$
F\ =\ \tfrac{1}{2}\Big(\Phi'\big(\psi(x)\big)\, I_\alpha\, e^\psi_{\ \mu} e^\alpha_{\ \nu}\ -\ \tfrac{1}{2}\big(1{-}\Phi\big(\psi(x)\big)^2\big)\, I_\alpha\, f^\alpha_{\ \beta\gamma}\, e^\beta_{\ \mu} e^\gamma_{\ \nu}\Big)\,\mathrm{d}x^\mu\wedge\mathrm{d}x^\nu\ : \tag{20}
$$

Using $(\tau,\rho,\psi)\to(\tau,\chi,\theta)\to(t,r,\theta)$ for $R{=}1$ and abbreviations $x\cdot\mathrm{d}x := x_\mu \mathrm{d}x^\mu$ and $\varepsilon^0 := 1$, $\varepsilon^1=\varepsilon^2 := \varepsilon$, we can write the Minkowski one-forms in a compact form as,

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Color EM fields in terms of Riemann–Silberstein vector $\vec{S} := \vec{E}+\mathrm{i}\vec{B}$:

$$S_x = -\tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{2\big[ty{+}\mathrm{i}x(z{+}\mathrm{i})\big]I_0 + 2\varepsilon\big[xy{+}\mathrm{i}t(z{+}\mathrm{i})\big]I_1 + \varepsilon\big[t^2{-}x^2{+}y^2{+}(z{+}\mathrm{i})^2\big]I_2\Big\}$$

$$S_y = \tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{2\big[tx{-}\mathrm{i}y(z{+}\mathrm{i})\big]I_0 + \varepsilon\big[t^2{+}x^2{-}y^2{+}(z{+}\mathrm{i})^2\big]I_1 + 2\varepsilon\big[xy{-}\mathrm{i}t(z{+}\mathrm{i})\big]I_2\Big\}$$

$$S_z = \tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{\mathrm{i}\big[t^2{+}x^2{+}y^2{-}(z{+}\mathrm{i})^2\big]I_0 + 2\varepsilon\big[\mathrm{i}tx{-}y(z{+}\mathrm{i})\big]I_1 + 2\varepsilon\big[\mathrm{i}ty{+}x(z{+}\mathrm{i})\big]I_2\Big\}$$

The corresponding stress-energy tensor

$$T_{\mu\nu} \;=\; -\tfrac{1}{2g^2}\left(\delta_\mu{}^\rho\delta_\nu{}^\lambda\eta^{\sigma\tau} - \tfrac14\eta_{\mu\nu}\eta^{\rho\lambda}\eta^{\sigma\tau}\right)\mathrm{tr}\left(F_{\rho\sigma}F_{\lambda\tau}\right) \tag{21}$$

also takes a nice compact form as $\left(\epsilon := -\tfrac14\left((\Phi')^2 + (1-\Phi^2)^2\right)\right)$

$$\left(T_{\mu\nu}\right) \;=\; \frac{8}{g^2}\,\frac{\epsilon}{(\lambda^2+4z^2)^3}\begin{pmatrix} t_{\alpha\beta} & t_{\alpha 3} \\ t_{3\alpha} & t_{33}\end{pmatrix} \quad \text{with} \quad \begin{cases} t_{\alpha\beta} \;=\; -\eta_{\alpha\beta}(\lambda^2+4z^2) + 16x_\alpha x_\beta z^2\,, \\ t_{3\alpha} \;=\; t_{\alpha 3} \;=\; -8x_\alpha z\,(\lambda-3z^2)\,, \\ t_{33} \;=\; 3\lambda^2 - 4z^2(1+4\lambda-4z^2) \end{cases} \tag{22}$$

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The fields $\vec{E}, \vec{B}$ and $T_{\mu\nu}$ are singular at the intersection of $\lambda=0$ hyperbola $H^{1,2}$ and $z=0$-plane.

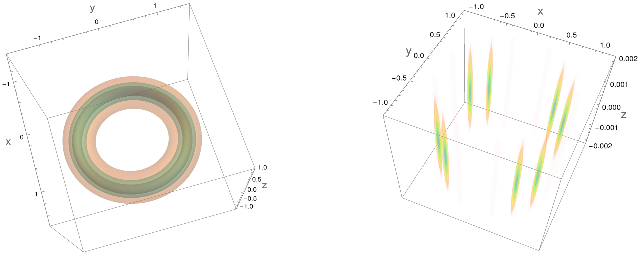

Figure 4: Plots for the energy density $\rho \propto (\lambda^2+4z^2)^{-2}|_{t=0}$. Left: Level sets for the values 10 (orange), 100 (cyan), and 1000 (brown). Right: Density plot emphasizing the maxima.

Abelian solution

For the Abelian case we judiciously chose $h(\psi)$ to obtain same fields on both sides of $H^{1,2}$,

$$\begin{aligned} \widetilde{A} &= -\tfrac{1}{2}\cos 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^0_{\ \mu}\, \mathrm{d}x^\mu \qquad \text{and} \\ \widetilde{F} &= \big\{\sin 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^\psi_{\ \mu} e^0_{\ \nu} - \cos 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^1_{\ \mu} e^2_{\ \nu}\big\}\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \ . \end{aligned} \tag{23}$$

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RS-vector reminiscent of the Hopf–Ranãda knot

$$\vec{\widetilde{E}} + \mathrm{i}\vec{\widetilde{B}} = \frac{4\mathrm{e}^{2\mathrm{i}\psi_0}}{(\lambda+2\mathrm{i}z)^3} \begin{pmatrix} -2\,\big(t\,y + \mathrm{i}x\,(z+\mathrm{i})\big) \\ 2\,\big(t\,x - \mathrm{i}y\,(z+\mathrm{i})\big) \\ \mathrm{i}\,\big(t^2 + x^2 + y^2 - (z+\mathrm{i})^2\big) \end{pmatrix}$$

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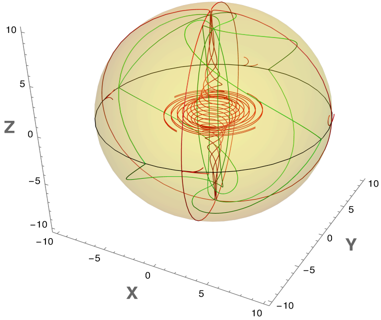
On the right we show typical electric (red) and magnetic (green) field lines for $t{=}10$ and $\psi_0{=}\frac{\pi}{2}$ inside the boundary sphere $r{=}\sqrt{101}$. The fields diverge at the (black) singular equatorial circle.



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Conclusion

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- Due to this singularity the total energy diverges for both kinds of gauge fields.
- Indeed, it is difficult to justify these solutions on physical grounds, but they could be relevant in some supergravity or syper Yang–Mills theories.

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THANK YOU!

Questions?