Exact gauge fields from anti-de Sitter space (arXiv:2301.03606)

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- In both cases, gluing of two copies of $\mathrm{dS}_4/\mathrm{AdS}_4$ is needed to cover full Minkowski space.

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Recap: $\mathrm{dS}_4 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$

- Two conformal avatars of dS_4 [KL20] with common S^2-metric $\mathrm{d}\Omega_2^2 = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2$ ($\theta \in [0,\pi]$ & $\phi \in [0,2\pi]$):

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- Effective map: $(t,r) \leftrightarrow (\tau,\chi)$ with constraint $\chi > |\tau|$:

$$\frac{t}{R} = \frac{\sin\tau}{\cos\tau - \cos\chi}\ , \quad \frac{r}{R} = \frac{\sin\chi}{\cos\tau - \cos\chi}\ . \tag{2}$$

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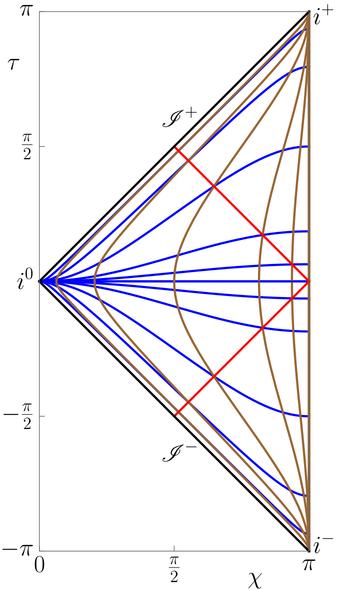

$\mathrm{AdS}_4 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$

- Consider following two conformal versions of AdS_4,

$$\begin{aligned} ds^2 &= \frac{R^2}{\cos^2\psi}\left(d\psi^2 - \cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\phi^2\right) \quad \mathrm{AdS_3\text{-}cyl} \\ &= \frac{R^2}{\cos^2\chi}\left(-d\tau^2 + d\chi^2 + \sin^2\chi\, d\Omega_2^2\right) \quad S^3_+\text{-cyl} \end{aligned} \tag{3}$$

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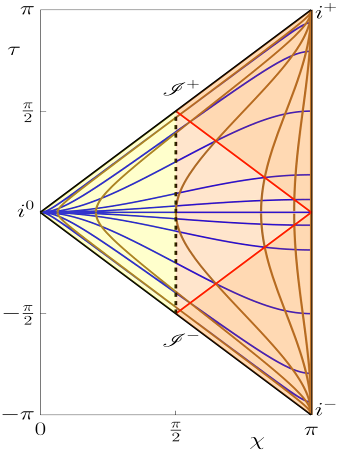
- Caveat: singularity encountered @boundary $\chi = \frac{\pi}{2}$, which is $r^2 - t^2 = R^2$.



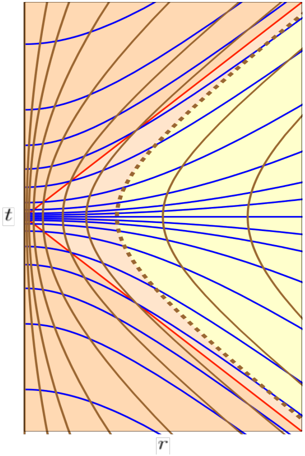


Figure 1: Gluing of two AdS_4 (across dashed line) to reveal full Minkowski space with the lightcone @origin (red). Left: Penrose diagram of (τ, χ) space with constant t- (blue) and r-slices (brown). Right: Minkowski space of (t, r) coordinates with constant (blue) τ- and (brown) χ-slices.

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SU(1, 1) Lie algebra and one-forms

$$\mathrm{AdS}_3 \ni (y^1, y^2, y^3, y^4) \mapsto \begin{pmatrix} y^1 - \mathrm{i}y^2 & y^3 - \mathrm{i}y^4 \\ y^3 + \mathrm{i}y^4 & y^1 + \mathrm{i}y^2 \end{pmatrix} \in \mathrm{SU}(1,1) \quad \text{group manifold} \tag{5}$$

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$$\Omega_L(g) \ = \ g^{-1}\mathrm{d}g \ = \ e^\alpha\, I_\alpha \ ; \quad [I_\alpha, I_\beta] \ = \ 2\, f^{\gamma}_{\ \alpha\beta}\, I_\gamma \quad \text{and} \quad \mathrm{tr}(I_\alpha\, I_\beta) \ = \ 2\, \eta_{\alpha\beta} \tag{6}$$

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$$\text{Cartan} \quad \mathrm{d}e^\alpha + f^\alpha_{\ \beta\gamma}\, e^\beta \wedge e^\gamma = 0 \quad \text{and} \quad \mathrm{d}s^2_{\mathrm{cyl}} = \mathrm{d}\psi^2 + \eta_{\alpha\beta}\, e^\alpha e^\beta \quad \text{ON-frame} \tag{7}$$

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A generic *gauge field* $\mathcal{A}$ in this frame can be made SU(1,1)-symmetric by,

$$\begin{aligned} \mathcal{A} \;&=\; \mathcal{A}_\psi\, e^\psi + \mathcal{A}_\alpha\, e^\alpha \quad \xrightarrow{\mathcal{A}_\psi = 0} \quad \mathcal{A} \;=\; X_\alpha(\psi)\, e^\alpha \\ \Longrightarrow\; \mathcal{F} \;&=\; \mathrm{d}\mathcal{A} + \mathcal{A}\wedge\mathcal{A} \;=\; X_\alpha'\, e^\psi \wedge e^\alpha + \tfrac{1}{2}\big(-2f^{\alpha}_{\ \beta\gamma} X_\alpha + [X_\beta, X_\gamma]\big)\, e^\beta \wedge e^\gamma \;. \end{aligned} \tag{8}$$

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Gauss-law constraint $[X_\alpha, X_\alpha'] = 0$ of the eom $*\mathrm{d}*\mathcal{F}=0$ can be readily satisfied for

$$X_0 = \Theta_0(\psi)\, I_0\ ,\quad X_1 = \Theta_1(\psi)\, I_1\ ,\quad X_2 = \Theta_2(\psi)\, I_2\ . \tag{9}$$

The Yang–Mills Langrangian $\mathcal{L} = \frac{1}{4}\mathrm{tr}(\mathcal{F} \wedge *\mathcal{F})$ computes to,

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$$\Theta_0 = \Theta_1 = \Theta_2 \ =: \ \tfrac{1}{2}\big(1 + \Phi(\psi)\big)$$
$$\implies \Phi'' \ = \ 2\,\Phi\,(1 - \Phi^2) \ = \ -\frac{\partial V}{\partial \Phi}$$

The Yang–Mills Langrangian $\mathcal{L} = \frac{1}{4}\mathrm{tr}(\mathcal{F}\wedge *\mathcal{F})$ computes to,

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{4}\mathrm{tr}\mathcal{F}_{\psi\alpha}\mathcal{F}^{\psi\alpha} + \tfrac{1}{8}\mathrm{tr}\mathcal{F}_{\beta\gamma}\mathcal{F}^{\beta\gamma} \\ &= \tfrac{1}{2}\sum_{\alpha}(\Theta'_\alpha)^2 - 2\{(\Theta_1-\Theta_2\Theta_0)^2 + (\Theta_2-\Theta_0\Theta_1)^2 + (\Theta_0-\Theta_1\Theta_2)^2\} \; .\end{aligned} \tag{10}$$

Notice the discrete symmetry of the permutation group S_4, acting by permuting $\{\Theta_\alpha\}$ and flipping the sign of any two. Its maximal normal subgroups S_3 and D_8 yields *exact solutions*:

- **Non-equivariant Abelian ansatz:**

$$\Theta_0 = \mathrm{h}(\psi) \qquad \text{while} \qquad \Theta_1 = \Theta_2 = 0 \; ,$$
$$\Longrightarrow \; \mathrm{h}'' = -4\,\mathrm{h}$$

- **Equivariant non-Ablian ansatz:**

$$\Theta_0 = \Theta_1 = \Theta_2 \;=:\; \tfrac{1}{2}\big(1+\Phi(\psi)\big)$$
$$\Longrightarrow \; \Phi'' \;=\; 2\,\Phi\,(1-\Phi^2) \;=\; -\frac{\partial V}{\partial \Phi}$$

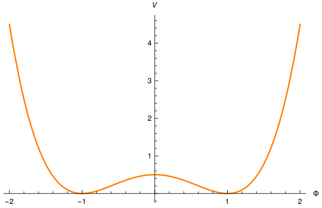

Figure 2: Potential $V(\Phi) = \frac{1}{2}(\Phi^2 - 1)^2$.

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Non-Abelian solution

Using $(\tau,\rho,\psi) \to (\tau,\chi,\theta) \to (t,r,\theta)$ for $R{=}1$ along with abbreviations $x \cdot \mathrm{d}x := x_\mu \mathrm{d}x^\mu$, $\varepsilon^1 = \varepsilon^2 := \varepsilon$ and $\varepsilon^0 := 1$ we can write the Minkowski one-forms in a compact form as,

$$\begin{aligned} e^\alpha &= \tfrac{\varepsilon^\alpha}{\lambda^2+4z^2}\left(2(\lambda{+}2)\,\mathrm{d}x^\alpha - 4x^\alpha\, x\cdot\mathrm{d}x - 4\, f^{\alpha}_{\ \beta\gamma}\, x^\beta \mathrm{d}x^\gamma\right)\ , \\ e^\psi &= \tfrac{\varepsilon}{\lambda^2+4z^2}\left(-2\lambda\,\mathrm{d}z + 4z\, x\cdot\mathrm{d}x\right)\ , \qquad \text{where} \quad \lambda \ := \ r^2 - t^2 - 1\ . \end{aligned} \tag{11}$$

Using $(\tau,\rho,\psi) \to (\tau,\chi,\theta) \to (t,r,\theta)$ for $R{=}1$ along with abbreviations $x\cdot \mathrm{d}x := x_\mu \mathrm{d}x^\mu$, $\varepsilon^1 = \varepsilon^2 := \varepsilon$ and $\varepsilon^0 := 1$ we can write the Minkowski one-forms in a compact form as,

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e^\psi &= \tfrac{\varepsilon}{\lambda^2+4z^2}\left(-2\lambda\,\mathrm{d}z + 4z\, x\cdot\mathrm{d}x\right)\ , \qquad \text{where} \quad \lambda := r^2 - t^2 - 1\ .
\end{aligned}
\tag{11}
$$

Fields are obtained from the field strength F of $\mathcal{A} \equiv A = \frac{1}{2}\Big(1+\Phi\big(\psi(x)\big)\Big)\, I_\alpha\ e^\alpha_{\ \mu}\ \mathrm{d}x^\mu$,

$$
F = \tfrac{1}{2}\Big(\Phi'\big(\psi(x)\big)\, I_\alpha\, e^\psi_{\ \mu} e^\alpha_{\ \nu} \ - \ \tfrac{1}{2}\big(1{-}\Phi\big(\psi(x)\big)^2\big)\, I_\alpha\, f^{\alpha}_{\ \beta\gamma}\, e^\beta_{\ \mu} e^\gamma_{\ \nu}\Big)\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu\ : \tag{12}
$$

Using $(\tau,\rho,\psi)\to(\tau,\chi,\theta)\to(t,r,\theta)$ for $R{=}1$ along with abbreviations $x\cdot\mathrm{d}x := x_\mu \mathrm{d}x^\mu$, $\varepsilon^1=\varepsilon^2:=\varepsilon$ and $\varepsilon^0:=1$ we can write the Minkowski one-forms in a compact form as,

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Color EM fields in terms of Riemann–Silberstein vector $\vec{S}:=\vec{E}+\mathrm{i}\vec{B}$:

$$S_x = -\tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{2\big[ty{+}\mathrm{i}x(z{+}\mathrm{i})\big]I_0 + 2\varepsilon\big[xy{+}\mathrm{i}t(z{+}\mathrm{i})\big]I_1 + \varepsilon\big[t^2{-}x^2{+}y^2{+}(z{+}\mathrm{i})^2\big]I_2\Big\}$$

$$S_y = \tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{2\big[tx{-}\mathrm{i}y(z{+}\mathrm{i})\big]I_0 + \varepsilon\big[t^2{+}x^2{-}y^2{+}(z{+}\mathrm{i})^2\big]I_1 + 2\varepsilon\big[xy{-}\mathrm{i}t(z{+}\mathrm{i})\big]I_2\Big\}$$

$$S_z = \tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{\mathrm{i}\big[t^2{+}x^2{+}y^2{-}(z{+}\mathrm{i})^2\big]I_0 + 2\varepsilon\big[\mathrm{i}tx{-}y(z{+}\mathrm{i})\big]I_1 + 2\varepsilon\big[\mathrm{i}ty{+}x(z{+}\mathrm{i})\big]I_2\Big\}$$

The corresponding stress-energy tensor

$$T_{\mu\nu} \;=\; -\tfrac{1}{2g^2}\left(\delta_\mu{}^\rho\delta_\nu{}^\lambda\eta^{\sigma\tau} - \tfrac{1}{4}\eta_{\mu\nu}\eta^{\rho\lambda}\eta^{\sigma\tau}\right)\mathrm{tr}\big(F_{\rho\sigma}F_{\lambda\tau}\big) \tag{13}$$

also takes a nice compact form as $\left(\epsilon := -\frac{1}{4}\big((\Phi')^2 + (1-\Phi^2)^2\big)\right)$

$$\big(T_{\mu\nu}\big) \;=\; \frac{8}{g^2}\,\frac{\epsilon}{(\lambda^2+4z^2)^3}\begin{pmatrix} t_{\alpha\beta} & t_{\alpha 3} \\ t_{3\alpha} & t_{33}\end{pmatrix} \quad \text{with} \quad \begin{cases} t_{\alpha\beta} \;=\; -\eta_{\alpha\beta}(\lambda^2+4z^2) + 16x_\alpha x_\beta z^2\,, \\ t_{3\alpha} \;=\; t_{\alpha 3} \;=\; -8x_\alpha z\,(\lambda-3z^2)\,, \\ t_{33} \;=\; 3\lambda^2 - 4z^2(1+4\lambda-4z^2) \end{cases} \tag{14}$$

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The fields $\vec{E}, \vec{B}$ and $T_{\mu\nu}$ are singular at the intersection of $\lambda=0$ hyperbola $H^{1,2}$ and $z=0$-plane.

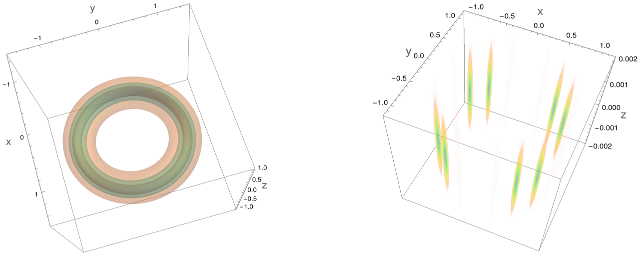

Figure 3: Plots for the energy density $\rho \propto (\lambda^2+4z^2)^{-2}|_{t=0}$. Left: Level sets for the values 10 (orange), 100 (cyan), and 1000 (brown). Right: Density plot emphasizing the maxima.

Abelian solution

Same field expression across the hyperbola $\lambda=0$; choose $\widetilde{A} \;=\; -\frac{1}{2}\cos 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^{0}{}_{\mu}\, \mathrm{d}x^{\mu}$:

$$\widetilde{F} \;=\; \big\{\sin 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^{\psi}{}_{\mu} e^{0}{}_{\nu} - \cos 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^{1}{}_{\mu} e^{2}{}_{\nu}\big\}\, \mathrm{d}x^{\mu} \wedge \mathrm{d}x^{\nu}\ . \tag{15}$$

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RS-vector reminiscent of the Hopf–Ranãda knot

$$\vec{\widetilde{E}} + \mathrm{i}\vec{\widetilde{B}} = \frac{4\mathrm{e}^{2\mathrm{i}\psi_0}}{(\lambda+2\mathrm{i}z)^3}\begin{pmatrix} -2\left(t\,y + \mathrm{i}x\,(z+\mathrm{i})\right) \\ 2\left(t\,x - \mathrm{i}y\,(z+\mathrm{i})\right) \\ \mathrm{i}\left(t^2+x^2+y^2-(z+\mathrm{i})^2\right)\end{pmatrix}$$

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- A limiting case $\psi_0{=}z{=}0$ reproduces the magnetic field of a magnetic vortex [RS18].

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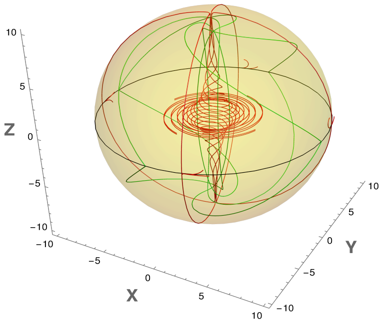
- Typical electric (red) and magnetic (green) field lines on right for $t{=}10$, $\psi_0{=}\frac{\pi}{2}$ inside $r{=}\sqrt{101}$. The fields diverge at the (black) singular equatorial circle: $\lambda{=}0 \bigcap z{=}0 \bigcap t{=}10$.



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- First, we employed the AdS_3-slicing of AdS_4 and the group manifold structure of $\mathrm{SU}(1,1)$ to find Yang–Mills solutions on $\mathcal{I} \times \mathrm{AdS}_3$.

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- Nevertheless, our Abelian solutions were found to match the magnetic field of a known vortex magnetic mode on $\mathrm{SU}(1,1)$ [RS18]. What about the other two Abelian solutions?

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THANK YOU!

Questions?