Exact gauge fields from anti-de Sitter space
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- For us the strategy would be: $\mathcal{I} \times \mathrm{AdS}_3 \xleftarrow{\text{conformal}} \mathrm{AdS}_4 \xrightarrow{\text{conformal}} \mathcal{I} \times S^3_+$ followed by the previous $\mathcal{I} \times S^3 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$ and noting that AdS_3 is the group manifold of SU(1,1).

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- In both cases, gluing of two copies of $\mathrm{dS}_4/\mathrm{AdS}_4$ is used to cover the full Minkowski space.

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Recap: $\mathrm{dS}_4 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$

It is well known that dS_4, seen an an embedding in $\mathbb{R}^{1,4}$ via

$$-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 \;=\; R^2 \;, \tag{1}$$

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$$\begin{aligned}\mathrm{d}s^2 \;&=\; \frac{R^2}{\sin^2\tau}\left(-\mathrm{d}\tau^2+\mathrm{d}\chi^2+\sin^2\chi\mathrm{d}\Omega_2^2\right) \;=\; \frac{R^2}{\sin^2\tau}\left(-\mathrm{d}\tau^2+\mathrm{d}\Omega_3^2\right)\\ &=\; \frac{R^2}{t^2}\left(-\mathrm{d}t^2+\mathrm{d}r^2+r^2\mathrm{d}\Omega_2^2\right) \;=\; \frac{R^2}{t^2}\left(-\mathrm{d}t^2+\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right)\end{aligned} \tag{2}$$

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where $\tau\in\mathcal{I}:=(-\pi/2,\pi/2)$, $\chi\in[0,\pi]$, $t\in\mathbb{R}_+$ & $r\in\mathbb{R}_+$. Notice that this only covers future half of the Minkowski space! This is due to the following constraint:

$$\cos\tau>\cos\chi \quad\Longleftrightarrow\quad \chi>|\tau|\;. \tag{3}$$

- S^2 coordinates $\theta \in [0, \pi]$ & $\phi \in [0, 2\pi]$ identified on both sides.

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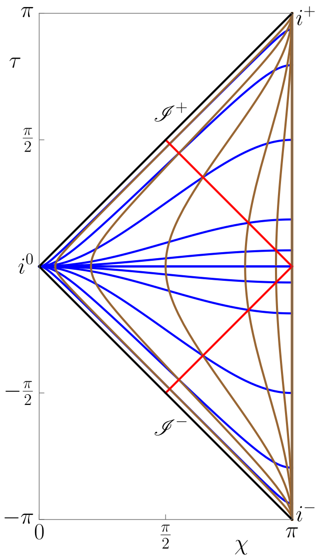
- This is clearly deomnstrated with the (τ,χ) Penrose diagram on the right with t- and r-slices.



$\mathrm{AdS}_4 \xrightarrow{\text{conformal}} \mathbb{R}^{1,3}$

We can isometrically embed AdS_4 inside $\mathbb{R}^{2,3}$ as

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Here too this takes two useful conformal avatars, easily seen from its flat-metric:

$$\begin{aligned} \mathrm{d}s^2 \;&=\; \frac{R^2}{\cos^2\psi}\big(\mathrm{d}\psi^2-\cosh^2\!\rho\,\mathrm{d}\tau^2+\mathrm{d}\rho^2+\sinh^2\!\rho\,\mathrm{d}\phi^2\big) \;=\; \frac{R^2}{\cos^2\psi}\big(\mathrm{d}\psi^2+\mathrm{d}\Omega^2_{1,2}\big) \\ &=\; \frac{R^2}{\cos^2\chi}\big(-\mathrm{d}\tau^2+\mathrm{d}\chi^2+\sin^2\!\chi\,\mathrm{d}\Omega^2_2\big) \;=\; \frac{R^2}{\cos^2\chi}\big(-\mathrm{d}\tau^2+\mathrm{d}\Omega^2_{3+}\big) \end{aligned} \tag{6}$$

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where $\psi\in\mathcal{I}\equiv(-\pi/2,\pi/2)$, $\rho\in\mathbb{R}_+$, $\phi,\tau\in S^1$, but $\chi\in[0,\pi/2]$. Thus, we get cylinders

(a) over (half of) the 3-sphere S^3, and

(b) over $\underline{\mathrm{AdS}_3\cong \mathrm{SU}(1,1)/\{\pm\mathrm{id}\}}$ with metric $\mathrm{d}\Omega_{1,2}$.

Here the S^3_+-structure is, in fact, nothing but a conformal compactification of H^3. To see this, let us change coordinates $(\rho,\psi)\to(\lambda,\theta)\in\mathbb{R}_+\times[0,\pi]$ in (6) via

$$\begin{aligned} \tanh\rho &= \sin\theta\,\tanh\lambda \qquad \text{and} \qquad \tan\psi = -\cos\theta\,\sinh\lambda \\ \Longrightarrow \quad \mathrm{d}s^2 &= R^2\big(-\cosh^2\lambda\,\mathrm{d}\tau^2+\mathrm{d}\lambda^2+\sinh^2\lambda\,\mathrm{d}\Omega_2^2\big)\ . \end{aligned} \tag{7}$$

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- Joining two H^3-slices while keeping τ fixed. Equivalently, this means gluing two S^3-hemispheres, namely S^3_+ and S^3_-, along the equatorial S^2:

$$\begin{aligned} &\tanh\rho=\sin\theta\,\tanh\lambda=\varepsilon\,\sin\theta\,\sin\chi \quad \& \quad \tan\psi=-\varepsilon\,\cos\theta\,\sinh\lambda=-\varepsilon\,\cos\theta\,\tan\chi \\ &\text{where}\ \begin{cases} \varepsilon=+1\ :\quad \rho,\lambda\in\mathbb{R}_+\,,\ \ \chi\in[0,\tfrac{\pi}{2}) \quad\Leftrightarrow\quad \text{northern hemisphere } S^3_+ \\ \varepsilon=-1\ :\quad \rho,\lambda\in\mathbb{R}_-\,,\ \ \chi\in(\tfrac{\pi}{2},\pi] \quad\Leftrightarrow\quad \text{southern hemisphere } S^3_- \end{cases} \end{aligned} \tag{8}$$

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- Use the previous $(\tau,\chi)\to(t,r)$ map to get to the Minkowski space.

This gluing, unlike in the de Sitter case, is not smooth and has singularity at the boundary:

$$\begin{aligned} &\left\{\psi = \pm \tfrac{\pi}{2}\right\} \;=\; \left\{\lambda = \pm\infty\right\} \;=\; \left\{\chi = \tfrac{\pi}{2}\right\} \\ \Longleftrightarrow \quad &\left\{r^2 - t^2 = R^2\right\} \;=:\; H_R^{1,2} \;\cong\; \mathrm{dS}_3 \;\cong\; \mathcal{I}_\tau \times S^2\big|_{\mathrm{bdy}} \;. \end{aligned} \tag{9}$$

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One needs to be careful of the orientation of two two copies at the boundary as shown below

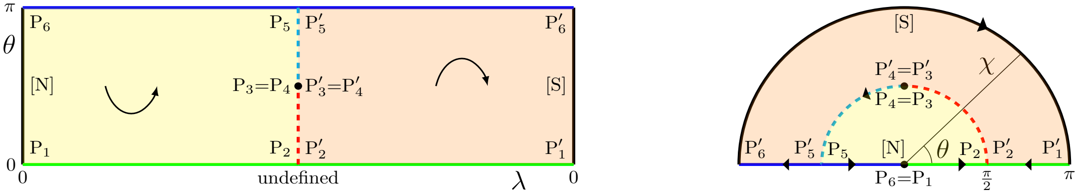

Figure 1: Gluing S_3^+ (yellow shaded region) to S_3^- (orange shaded region) along the (dashed) boundary.

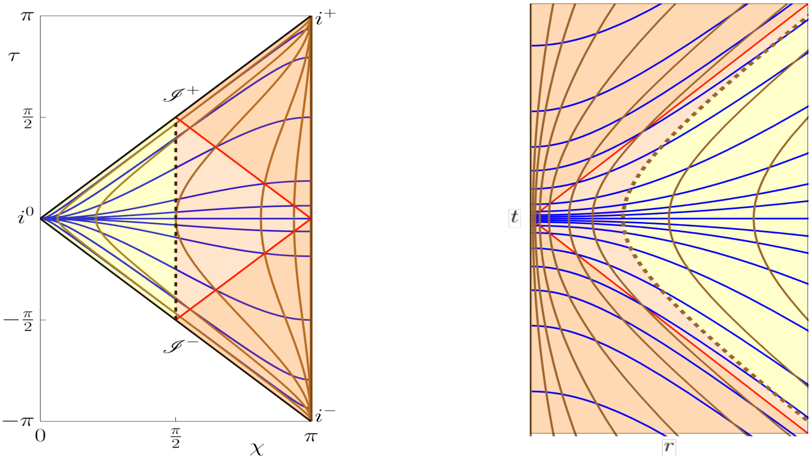

Figure 2: Gluing of two AdS_4 to reveal the full Minkowski space with the lightcone (red). Left: (τ, χ) AdS_4 space (two copies) yielding the Penrose diagram with constant t- (blue) and r-slices (brown). Right: (t, r) Minkowski space with boundary hyperbola $H_R^{1,2}$ (dashed) and constant τ- (blue) and χ-slices (brown).

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SU(1, 1) Lie algebra and one-forms

We start by noticing that AdS_3 is the group manifold of $\mathrm{SU}(1,1)$:

$$g:\ \mathrm{AdS}_3 \to \mathrm{SU}(1,1) \qquad \text{via} \qquad (y^1,y^2,y^3,y^4) \mapsto \begin{pmatrix} y^1-\mathrm{i}y^2 & y^3-\mathrm{i}y^4 \\ y^3+\mathrm{i}y^4 & y^1+\mathrm{i}y^2 \end{pmatrix}\,. \tag{10}$$

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This map also yields the left-invariant one-forms e^α, $\alpha = 0,1,2$ via Maurer–Cartan method:

$$\Omega_L(g) \;=\; g^{-1}\mathrm{d}g \;=\; e^\alpha I_\alpha\ ; \quad I_0 \;=\; \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix} ,\ I_1 \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,\ I_2 \;=\; \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} , \tag{11}$$

where the $\mathfrak{sl}(2,\mathbb{R})$ generators I_α of $\mathrm{PSL}(2,\mathbb{R}) \cong \mathrm{SU}(1,1)/\{\pm\mathrm{id}\}$ are subject to

$$[I_\alpha, I_\beta] \;=\; 2\, f^\gamma_{\ \alpha\beta}\, I_\gamma \qquad \text{and} \qquad \mathrm{tr}(I_\alpha\, I_\beta) \;=\; 2\,\eta_{\alpha\beta} \qquad \text{with} \quad (\eta_{\alpha\beta}) = \mathrm{diag}(-1,1,1)\ , \tag{12}$$

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and $f^2_{\ 01} = f^1_{\ 20} = -f^0_{\ 12} = 1$. Explicitly, we get the following one-forms on AdS_3

$$\begin{aligned} e^0 &= \cosh^2\!\rho\ \mathrm{d}\tau + \sinh^2\!\rho\ \mathrm{d}\phi\ , \\ e^1 &= \cos(\tau-\phi)\ \mathrm{d}\rho + \sinh\rho\ \cosh\rho\ \sin(\tau-\phi)\ \mathrm{d}(\tau+\phi)\ , \\ e^2 &= -\sin(\tau-\phi)\ \mathrm{d}\rho + \sinh\rho\ \cosh\rho\ \cos(\tau-\phi)\ \mathrm{d}(\tau+\phi)\ . \end{aligned} \tag{13}$$

The (non-)equivariant ansatz

These one-forms e^α satisfy following Cartan structure equation

$$\mathrm{d}e^\alpha + f^{\alpha}_{\ \beta\gamma}\, e^\beta \wedge e^\gamma = 0 \tag{14}$$

and provide a local orthonormal frame on the cylinder $\mathcal{I} \times \mathrm{AdS}_3$:

$$\mathrm{d}s^2_{\mathrm{cyl}} \ = \ \mathrm{d}\psi^2 + \eta_{\alpha\beta}\, e^\alpha e^\beta \ =: \ (e^\psi)^2 - (e^0)^2 + (e^1)^2 + (e^2)^2 \ . \tag{15}$$

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A generic gauge field in this frame is given by

$$\mathcal{A} \;=\; \mathcal{A}_\psi\, e^\psi + \mathcal{A}_\alpha\, e^\alpha \ , \tag{16}$$

which can be made SU(1,1)-symmetric by setting $\mathcal{A}_\psi = 0$, so that $\mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$ becomes

$$\mathcal{A} \;=\; X_\alpha(\psi)\, e^\alpha \qquad \Longrightarrow \qquad \mathcal{F} \;=\; X_\alpha'\, e^\psi \wedge e^\alpha + \tfrac{1}{2}\big(-2f^{\alpha}_{\ \beta\gamma} X_\alpha + [X_\beta, X_\gamma]\big)\, e^\beta \wedge e^\gamma \ . \tag{17}$$

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Moreover, to satisfy the Gauss-law constraint $[X_\alpha, X'_\alpha] = 0$ (from eom: $*\mathrm{d} * \mathcal{F} = 0$) we chose

$$X_0 = \Theta_0(\psi)\, I_0 \ , \quad X_1 = \Theta_1(\psi)\, I_1 \ , \quad X_2 = \Theta_2(\psi)\, I_2 \ . \tag{18}$$

This gives us the following Yang–Mills Langrangian

$$
\begin{aligned}
\mathcal{L} &= \tfrac{1}{4}\mathrm{tr}\mathcal{F}_{\psi\alpha}\mathcal{F}^{\psi\alpha} + \tfrac{1}{8}\mathrm{tr}\mathcal{F}_{\beta\gamma}\mathcal{F}^{\beta\gamma} \\
&= \tfrac{1}{2}\big\{(\Theta_1')^2 + (\Theta_2')^2 + (\Theta_0')^2\big\} \ - \ 2\big\{(\Theta_1-\Theta_2\Theta_0)^2 + (\Theta_2-\Theta_0\Theta_1)^2 + (\Theta_0-\Theta_1\Theta_2)^2\big\}\ ,
\end{aligned}
\tag{19}
$$

which enjoys a discrete symmetry of the permutation group S_4, which acts by permuting three fields and flipping the sign of any two fields.

This gives us the following Yang–Mills Langrangian

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{4}\mathrm{tr}\mathcal{F}_{\psi\alpha}\mathcal{F}^{\psi\alpha} + \tfrac{1}{8}\mathrm{tr}\mathcal{F}_{\beta\gamma}\mathcal{F}^{\beta\gamma} \\ &= \tfrac{1}{2}\big\{(\Theta_1')^2 + (\Theta_2')^2 + (\Theta_0')^2\big\} \;-\; 2\big\{(\Theta_1-\Theta_2\Theta_0)^2 + (\Theta_2-\Theta_0\Theta_1)^2 + (\Theta_0-\Theta_1\Theta_2)^2\big\}\ ,\end{aligned} \tag{19}$$

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- **Non-equivariant Abelian ansatz:**

$$\Theta_0 = \mathrm{h}(\psi) \qquad \text{while} \qquad \Theta_1 = \Theta_2 = 0\;,$$
$$\Longrightarrow\; \mathrm{h}'' = -4\,\mathrm{h}$$

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- **Equivariant non-Ablian ansatz:**

$$\Theta_0 = \Theta_1 = \Theta_2 \;=:\; \tfrac{1}{2}\big(1+\Phi(\psi)\big)$$
$$\Longrightarrow \; \Phi'' \;=\; 2\,\Phi\,(1-\Phi^2) \;=\; -\frac{\partial V}{\partial\Phi}$$

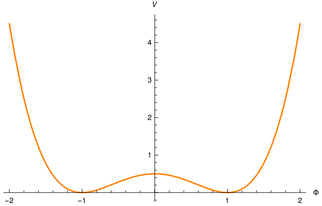

Figure 3: Potential $V(\Phi) = \frac{1}{2}(\Phi^2-1)^2$.

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Non-Abelian solution

Using $(\tau,\rho,\psi)\rightarrow(\tau,\chi,\theta)\rightarrow(t,r,\theta)$ for $R{=}1$ along with abbreviations $x\cdot\mathrm{d}x := x_\mu \mathrm{d}x^\mu$, $\varepsilon^1=\varepsilon^2:=\varepsilon$ and $\varepsilon^0:=1$ we can write the Minkowski one-forms in a compact form as,

$$
\begin{aligned}
e^\alpha &= \tfrac{\varepsilon^\alpha}{\lambda^2+4z^2}\left(2(\lambda{+}2)\,\mathrm{d}x^\alpha - 4x^\alpha\, x\cdot\mathrm{d}x - 4\,f^{\alpha}_{\ \beta\gamma}\,x^\beta \mathrm{d}x^\gamma\right)\ , \\
e^\psi &= \tfrac{\varepsilon}{\lambda^2+4z^2}\left(-2\lambda\,\mathrm{d}z + 4z\,x\cdot\mathrm{d}x\right)\ , \qquad \text{where} \quad \lambda \ := \ r^2 - t^2 - 1\ .
\end{aligned}
\tag{20}
$$

Using $(\tau,\rho,\psi)\to(\tau,\chi,\theta)\to(t,r,\theta)$ for $R{=}1$ along with abbreviations $x\cdot \mathrm{d}x := x_\mu \mathrm{d}x^\mu$, $\varepsilon^1=\varepsilon^2:=\varepsilon$ and $\varepsilon^0:=1$ we can write the Minkowski one-forms in a compact form as,

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Fields are obtained from the field strength F of $\mathcal{A}\equiv A=\frac{1}{2}\Big(1+\Phi\big(\psi(x)\big)\Big)\, I_\alpha\, e^\alpha_{\ \mu}\,\mathrm{d}x^\mu$,

$$F \;=\; \tfrac{1}{2}\Big(\Phi'\big(\psi(x)\big)\, I_\alpha\, e^\psi_{\ \mu} e^\alpha_{\ \nu} \;-\; \tfrac{1}{2}\big(1{-}\Phi\big(\psi(x)\big)^2\big)\, I_\alpha\, f^\alpha_{\ \beta\gamma}\, e^\beta_{\ \mu} e^\gamma_{\ \nu}\Big)\,\mathrm{d}x^\mu\wedge\mathrm{d}x^\nu : \tag{21}$$

Using $(\tau,\rho,\psi)\rightarrow(\tau,\chi,\theta)\rightarrow(t,r,\theta)$ for $R{=}1$ along with abbreviations $x\cdot\mathrm{d}x := x_\mu \mathrm{d}x^\mu$, $\varepsilon^1=\varepsilon^2:=\varepsilon$ and $\varepsilon^0:=1$ we can write the Minkowski one-forms in a compact form as,

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Fields are obtained from the field strength F of $\mathcal{A}\equiv A=\frac12\Big(1+\Phi\big(\psi(x)\big)\Big)\, I_\alpha\, e^\alpha_{\ \mu}\,\mathrm{d}x^\mu$,

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Color EM fields in terms of Riemann–Silberstein vector $\vec{S}:=\vec{E}+\mathrm{i}\vec{B}$:

$$S_x = -\tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{2\big[ty+\mathrm{i}x(z{+}\mathrm{i})\big]I_0 + 2\varepsilon\big[xy+\mathrm{i}t(z{+}\mathrm{i})\big]I_1 + \varepsilon\big[t^2-x^2+y^2+(z{+}\mathrm{i})^2\big]I_2\Big\}$$

$$S_y = \tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{2\big[tx-\mathrm{i}y(z{+}\mathrm{i})\big]I_0 + \varepsilon\big[t^2+x^2-y^2+(z{+}\mathrm{i})^2\big]I_1 + 2\varepsilon\big[xy-\mathrm{i}t(z{+}\mathrm{i})\big]I_2\Big\}$$

$$S_z = \tfrac{2(\mathrm{i}\varepsilon\Phi'+\Phi^2-1)}{(\lambda-2\mathrm{i}z)(\lambda+2\mathrm{i}z)^2}\Big\{\mathrm{i}\big[t^2+x^2+y^2-(z{+}\mathrm{i})^2\big]I_0 + 2\varepsilon\big[\mathrm{i}tx-y(z{+}\mathrm{i})\big]I_1 + 2\varepsilon\big[\mathrm{i}ty+x(z{+}\mathrm{i})\big]I_2\Big\}$$

The corresponding stress-energy tensor

$$T_{\mu\nu} \;=\; -\tfrac{1}{2g^2}\left(\delta_\mu{}^\rho\delta_\nu{}^\lambda\eta^{\sigma\tau} - \tfrac{1}{4}\eta_{\mu\nu}\eta^{\rho\lambda}\eta^{\sigma\tau}\right)\mathrm{tr}\big(F_{\rho\sigma}F_{\lambda\tau}\big) \tag{22}$$

also takes a nice compact form as $\left(\epsilon := -\frac{1}{4}\big((\Phi')^2 + (1-\Phi^2)^2\big)\right)$

$$\big(T_{\mu\nu}\big) \;=\; \frac{8}{g^2}\,\frac{\epsilon}{(\lambda^2+4z^2)^3}\begin{pmatrix} \mathrm{t}_{\alpha\beta} & \mathrm{t}_{\alpha 3} \\ \mathrm{t}_{3\alpha} & \mathrm{t}_{33}\end{pmatrix} \quad \text{with} \quad \begin{cases} \mathrm{t}_{\alpha\beta} \;=\; -\eta_{\alpha\beta}(\lambda^2+4z^2) + 16x_\alpha x_\beta z^2\,, \\ \mathrm{t}_{3\alpha} \;=\; \mathrm{t}_{\alpha 3} \;=\; -8x_\alpha z\,(\lambda-3z^2)\;, \\ \mathrm{t}_{33} \;=\; 3\lambda^2 - 4z^2(1+4\lambda-4z^2) \end{cases} \tag{23}$$

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The fields $\vec{E}, \vec{B}$ and $T_{\mu\nu}$ are singular at the intersection of $\lambda{=}0$ hyperbola $H^{1,2}$ and $z{=}0$-plane.

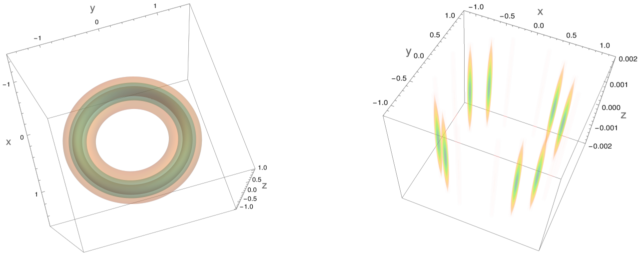

Figure 4: Plots for the energy density $\rho \propto (\lambda^2{+}4z^2)^{-2}|_{t=0}$. Left: Level sets for the values 10 (orange), 100 (cyan), and 1000 (brown). Right: Density plot emphasizing the maxima.

Abelian solution

For the Abelian case we judiciously chose $h(\psi)$ to obtain same fields on both sides of $H^{1,2}$,

$$\begin{aligned}\widetilde{A} &= -\tfrac{1}{2}\cos 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^0{}_\mu\, \mathrm{d}x^\mu \qquad \text{and} \\ \widetilde{F} &= \big\{\sin 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^\psi{}_\mu e^0{}_\nu - \cos 2\big(\psi(x)+\varepsilon\psi_0\big)\, e^1{}_\mu e^2{}_\nu\big\}\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \ .\end{aligned} \tag{24}$$

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RS-vector reminiscent of the Hopf–Ranãda knot

$$\vec{\widetilde{E}} + \mathrm{i}\vec{\widetilde{B}} = \frac{4\mathrm{e}^{2\mathrm{i}\psi_0}}{(\lambda+2\mathrm{i}z)^3}\begin{pmatrix} -2\,\big(t\,y+\mathrm{i}x\,(z+\mathrm{i})\big) \\ 2\,\big(t\,x-\mathrm{i}y\,(z+\mathrm{i})\big) \\ \mathrm{i}\,\big(t^2+x^2+y^2-(z+\mathrm{i})^2\big) \end{pmatrix}$$

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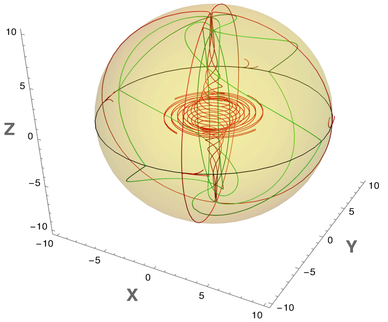
On the right we show typical electric (red) and magnetic (green) field lines for $t{=}10$ and $\psi_0{=}\frac{\pi}{2}$ inside the boundary sphere $r{=}\sqrt{101}$. The fields diverge at the (black) singular equatorial circle.



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Conclusion

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- Similar looking solutions were also found before using vortex equations and Dirac fields on AdS_4 [RS18]; it would be interesting to see how these solutions are related.

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THANK YOU!

Questions?